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Analysis Zones of Damage in the Interface Matrix Fibre

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Abstract - This article is devoted to the study of the decoherence of the interface (matrix fibre) in a heterogeneous material in which we analyze the evolution of separation. The problem consists, in this case, studying the behavior of the crack locally. This local study is more significant as the parameters influencing in the mechanisms of rupture can be determined in the methods of composition and development of heterogeneous materials. In addition, it gives us the opportunity to present a numerical technique using the integral equations of borders and to show its performances compared to the finite element method. We note that the use of the finite element method does not enable us to reach this precision, for the reasons related to the refinement of the grid. Generally, the analytical solutions compare satisfactorily well to the cohesive finite element calculations and experimental data in the literature. The interest that we carried to the comparison of the displacements obtained by the two methods has as an aim the validation of our results, in order to go up with the constraints meadows of the point of crack and to be able to calculate the factors of stress intensity. We noted that the examination on a small scale of the deformation at a peak of crack shows the high degree of accuracy in which the grid by elements of border allows compared with the grid by finite elements that requires to use elements of more reduced size.

Keywords: *cohesive model . surfaces and interfaces , Damage analyse , finite element*

1. Introduction

This work is devoted to study one of the principal tasks in developing crack criteria in long fibre reinforced composites material.

These composite materials must show excellent thermal conductivities as well as they should be able to keep its original shape and strength when subjected to thermal and mechanical loads. Composite materials are used extensively in aerospace and more recently in automotive applications. Their high mechanical performances and good fatigue durability offer definite advantages compared to more traditional materials.

The interfacial area between the fiber and matrix has a prime importance in the characterization of composite materials and their performance, as it ensures the load transfer between the fibers and matrix and provides materials with a high mechanical performance [15].
The stress and strain concentration can occur near the interface between the fibers and the matrix and cause damages such as interface debonding, formation of micro voids or micro cracks in matrix. Because the occurrence of these damages are depeudent greatly on micro stress and strain fields, thus the rnicromechanics is of special interest in mechanic analysis of composites [20]. In composite materials, the damage is completely different; this is due to the heterogeneity of the material, as the matrix and fibers have different mechanical behaviors. The distribution of the micro-strains induced by a total load of traction of one-way composite results from a synergy between fibres and the matrix. Indeed, the matrix is not used solely for dependent fibres between them, but it also takes part in the rigidity of the whole of the composite. It protects fibres from external pressures and deteriorations likely to start possible cracking . The load transfer mechanism between the matrix and the fibre interface behave an important role in determining the mechanical properties of fibre-reinforced composites [12, 14, 20, 24]. Performance of the composite materials depends on a good adhesion at the interface, between fibre and matrix.

The studies which we already carried out showed that the concentration and the discontinuity of these micro-strains at the zone of interface [1, 2, 3, 4]. Moreover, it is well-known that the basic components (matrix - fibres) often present defects of cracking created during the development or by damage under the effect of the external requests. This phenomenon explains the fact that the interfaces between the components exploit a paramount role the macroscopic mechanical behavior of the composites. It then becomes necessary to carry out a local study allowing the analysis of the mechanical behavior of these zones, where the concentrations of the constraints can be critical with respect to the resistance of the composite structures.

It poses the problems of singularities of constraints initiating the progression of the crack. It is well understood that the progression of the crack depends on the way in which the constraints decreased in the vicinity immediate of the singularity. The problem consists, in this case, studying the behavior of the crack locally. This local study is more significant as the parameters influencing in the mechanisms of rupture can be determined in the methods of composition and development of heterogeneous materials. This work is devoted to study the decoherence of the interface (matrix fibre) in a heterogeneous material in which we analyze the evolution of separation [13, 16, 21, 23]. Also we note good performance of the composite materials depends on a good adhesion at the interface, between fibre and matrix. In addition, it gives us the opportunity to present a numerical technique using the integral equations of borders and to show its performances compared to the finite element method.

2. Method of the Equations of Borders

While placing oneself within the framework of linear elasticity, we considers a medium Ω made up of *n* under - field Ω_i , presumed homogeneous. Each under - field can be delimited by one or several contours Γ_i , as in the case of under field Ω_3 of (fig. 1).

The basic idea around which the method of the equations of borders articulated is to be able to associate integrals of borders integrals of volumes [5, 6, 7,19, 25].

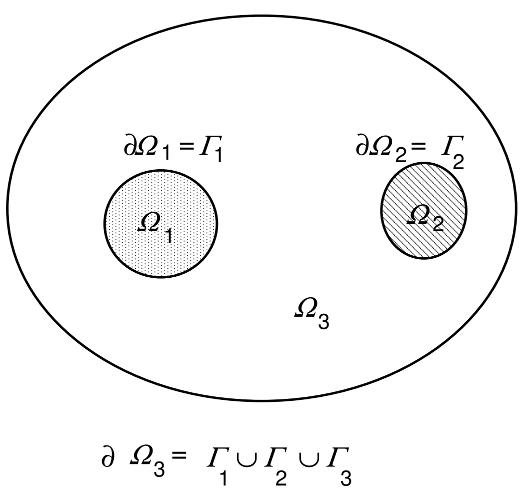

Fig. 1 Geometry of the heterogeneous structure.

we bases oneself on the theorem of reciprocal work of *Betti* who stipulates that in the same elastic system, the work of the forces external of a state of equilibrium defined by $\bar{u}, \bar{t}, \bar{f}$ in the field of displacements of a second state of equilibrium defined by $\bar{u}^*, \bar{t}^*, \bar{f}^*$ is equal to the work of the forces external of the second state of equilibrium in the field of displacements of the first state of equilibrium.

The translation of this theorem is thus written:

$$\int_\Omega (\bar{f}.\bar{u}^* - \bar{f}^*.\bar{u})dV + \int_\Gamma (\bar{t}.\bar{u}^* - \bar{t}^*.\bar{u})d\Gamma = 0 \qquad (1)$$

$\bar{u}, \bar{t}, \bar{f}$ are the fields of displacements, surface stresses and forces of volume of the first state of equilibrium.

$\bar{u}^*, \bar{t}^*, \bar{f}^*$ fields of displacements, surface stresses and forces of volume of the second state of equilibrium.

Ω the field occupied by the springy medium and Γ its contour.

By choosing for the first real state the real fields $\bar{u}, \bar{t}, \bar{f}$ and for the second state displacements $\bar{u}^*$ and tensions $\bar{t}^*$ corresponding, in an infinite medium, with a unit force 1. $\bar{e}^*$ and by introducing the function of influence of Green [8] to translate the influence of a unit force applied into a point P to displacements of another point **Q** :

$u_i(P) = U_{ij}(P,Q)e_j(Q)$ (U_{ij} the tensor of *Grenn represents*) (2)

The relation (1) translating the theorem of Betti is written, then:

$$\int_\Omega (U_{ij}(P,Q)f_i(Q) - \delta_{ij}\delta(P,Q)u_i(P)e_j^*)dV(Q) =$$

$$\int_\Gamma (T_{ij}(P,Q)u_i(P)e_j^*) - U_{ij}(P,Q)t_i(Q)d\Gamma \qquad (3)$$

In this relation $\delta(P,Q)$ the symbol of Dirac represents:

$$\delta(P,Q) = 0 \qquad \text{if } P \neq Q$$

$$\delta(P,Q) = 1 \qquad \text{if } P = Q$$

$$\int_{\Omega=\infty} \delta(P,Q)dV = 1$$

Thus, by neglecting the forces of volume, the identity of *Somigliana* makes it possible to show that the fields of displacements u and constraints σ in a point P interior with the field Ω are expressed according to the integral utilizing the fields of displacements $\bar{u}$ and tensions $\bar{t}$ on contour Γ:

$$u_i(P) = \int_\Gamma (U_{ij}(P,Q).\bar{t}_j(Q) - T_{ij}(P,Q).\bar{u}_j(Q))d\Gamma \qquad (4.a)$$

$$\sigma_{ij}(P) = \int_\Gamma (D_{ijk}(P,Q).\bar{t}_k(Q) - F_{ijk}(P,Q).\bar{u}_k(Q))d\Gamma \qquad (4.b)$$

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where Γ can be consisted of the meeting of several contours Γ_i and U_{ij} T_{ij} , D_{ijk} and F_{ijk} are known matrices. While being interested in the point of cracking, we makes tighten the point **P** towards the border Γ, as in (fig. 2).
We shows [9]:

$$\lim_{r\to 0}\int_{\Gamma} U_{ij}(P,Q).\bar{t}_j(Q)d\Gamma = 0 \qquad (5.a)$$

$$\lim_{r\to 0}\int_{\Gamma} T_{ij}(P,Q).\bar{t}_j(Q)d\Gamma = \alpha_{ij}u_j(P) \qquad (5.b)$$

α_{ij} represent the components given explicitly according to the angles of the border meadows of the crack.

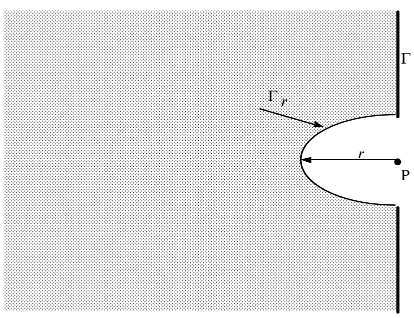

Fig. 2 point P in the vicinity of the crack.

Implementation the numerical requires the use of the functions of forms in an intrinsic frame of reference ξ ($0 \le \xi \le 1$), independently for the fields of displacements and tensions at the borders. Consequently, we associates each node *n* of each element *e* two unknown factors in displacements $\bar{u}$ and or tensions $\bar{t}$.
Thus, by using the numerical techniques of integrations, we leads to the system of equations:

$$[A]\{\bar{u}\} = [B]\{\bar{t}\} \qquad (6)$$

with :

$$A_{ij}^{ne}(P) = \int_0^1 T_{ij}(P, s(\xi)).J^e(\xi).N^n(\xi).d\xi \qquad (7.a)$$

$$B_{ij}^{ne}(P) = \int_0^1 U_{ij}(P, s(\xi)).J^e(\xi).N^n(\xi).d\xi \qquad (7.b)$$

The calculation of the integrals is accompanied by certain difficulties related to the presence of the singularities in the expressions of $\bar{u}$ and $\bar{t}$. It should be noted that the continuity of the tensions is not ensured at the time of the passage of an element another and there can be unfolding of tensions in the singular points. Moreover, the matrices *[A]* and *[B]* are not symmetrical and the matrix *[B]* is not necessarily square.
By locating the terms of displacements and tensions known and by taking account of the possible unfolding of tensions, we can overcome his difficulties and we writes the final system of linear equations to solve:
[L] {X} = {X}
where the matrix *[L]* is square and the vector *{X}* and composed of the tensions and unknown factors displacements of the problem

3. Application

The exposed results relate to the modelling of the separation of a circular heterogeneity of form and small size in front of that of the matrix (fig. 3).

It is supposed that part of the interface between this heterogeneity and the matrix is not perfectly adherent so that the crack forms an arc of angle equal to 2α (with α= 45. This crack represents beginning of cohesiveness in fibre and the matrix.

(Fig. 4) Represents two different grids. The first grid is useful for the implementation of the method of the integral equations of borders. Count with the second grid, it necessary to calculations by the finite element method.

(Fig. 5) Presents a confrontation of the results relating to the displacements, obtained by the two methods, in the vicinity of the point of crack. On the other hand, we note a good agreement between the two methods. In addition, we immediately note the advantage of the method of the integral equations of borders compared to that of the finite elements.

This advantage is at the level of the grid and its tiresome character, at the time of the implementation of the finite element method. Indeed, the refinement of the finite elements in the vicinity of the point of crack posed to us considerable problems when designing grid .

However, it is noted that the implementation of the method of the integral equations of borders requires a detailed attention when it is a question of overcoming the difficulties related to the presence of the singularities in the expressions of the fields of displacements and tensions, as well as the elimination of the unfolding of the tensions at the singular points.
The examination of the nodes of the grid, of the method of the integral equations of borders, to locate the unfolding of the tensions required the introduction into the software of a

module having for role the elimination of the useless points and the transformation of the matrices [A] and [B] nonsquare , in order to lead to a linear final system that we fear of solving by the traditional method of Gauss .

The interest that we carried to the comparison of the displacements obtained by the two methods has as an aim the validation of our results, in order to go up with the constraints meadows of the point of crack and to be able to calculate the stress intensity factors.

These factors will be defined like reference mark of evaluation of the rupture of material. We noted that the examination on a small scale of the deformation at a peak of crack shows the high degree of accuracy which the grid by elements of border allows, compared with the grid by finite elements which requires using elements of more reduced size.

This justifies the use of the results obtained by elements of borders in the evaluation of the stress intensity factors. In addition, the use of the factors of constraints as rupture locates comes owing to the fact that we cannot reason by the constraints, because these last become very high, and even infinite, when *r* tends towards zero. Indeed, the determination of the stress intensity factors K_I and K_{II} characteristic of the singularity are well-known analytically, in the case of a crack in arc of circle in a homogeneous material [10, 11, 15,17, 18, 22].

The latter can be deduced by approximating the constraints and displacements in the vicinity of the lips from the crack, by using the polar, local reference mark governed by the ray *r* and the angle θ (*r* indicates the distance between the point of calculation and the point of crack).

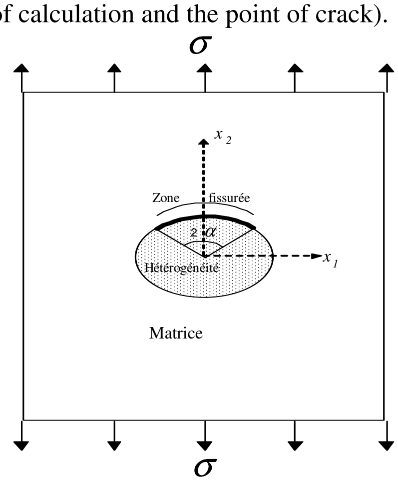

Fig. 3 Localization of the separated part of the interface enters heterogeneity and the matrix.

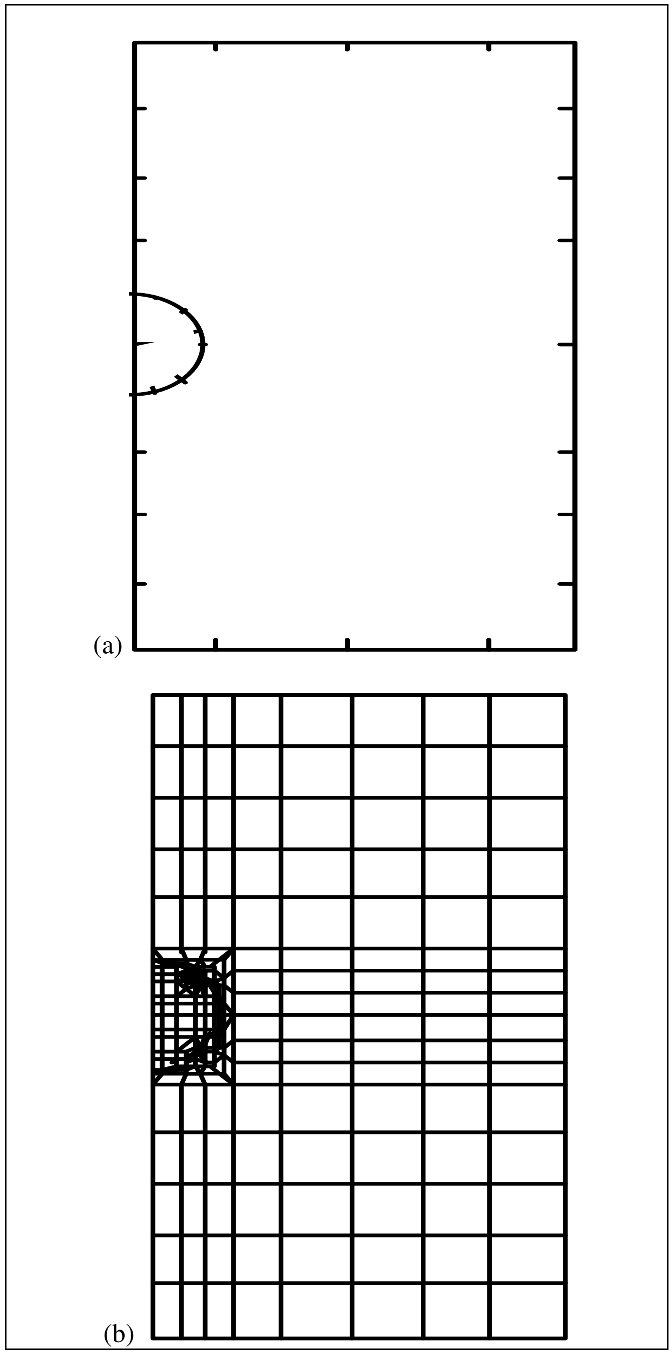

Fig. 4 (a) Grid of the integral equations - (b) Grid of the finite elements.

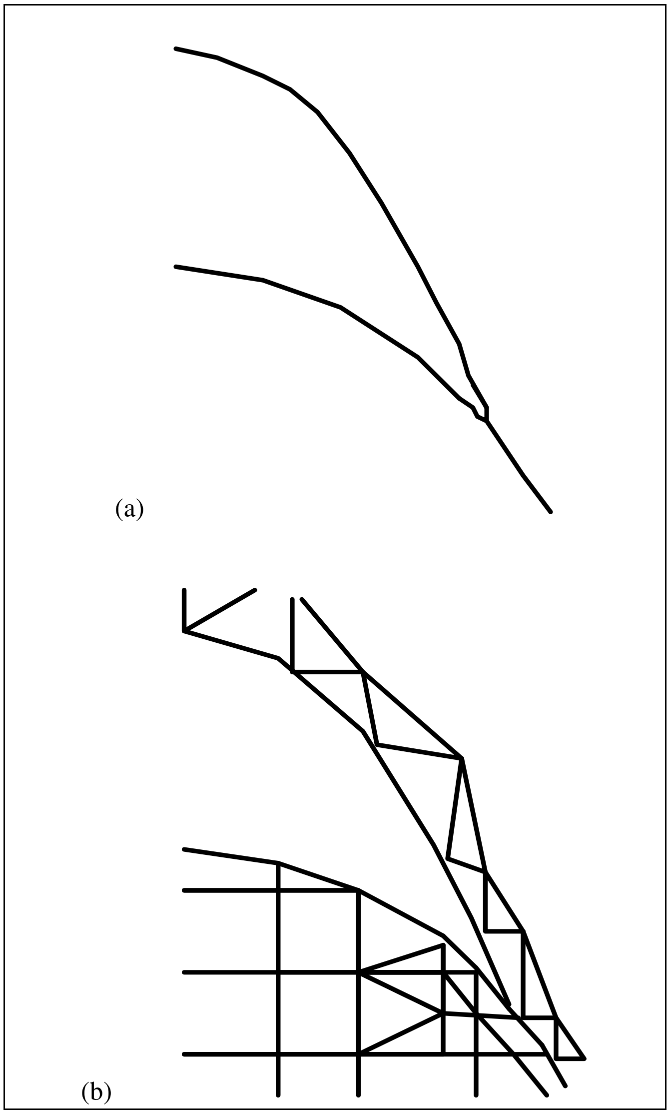

Fig. 5 Displacements resulting from the integral equations (a) and Finite element (b).

(Fig. 6) shows the evolution of the stress intensity factors according to the angle of the crack . The results obtained by the numerical method of the integral of borders, compared with the analytical results show a satisfactory precision. We note that the use of the finite element method does not have enabled to reach this precision, for reasons related to the refinement of the grid, as we announced.

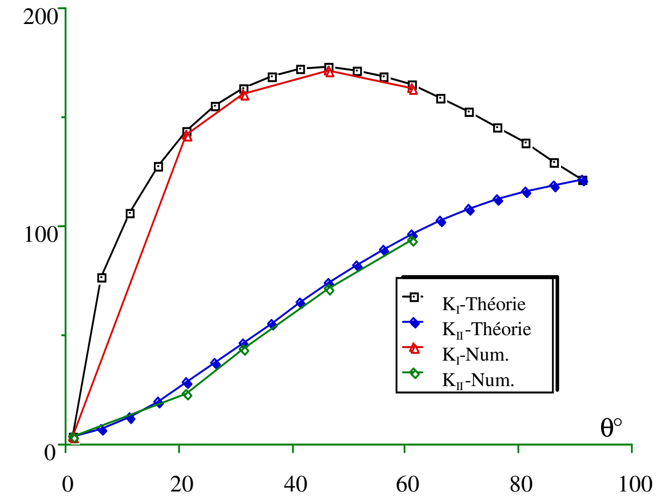

Fig. 6 Evolution of the stress intensity factors according to the angle of cracking θ (Unit $MP_a\sqrt{m}$).

4. Conclusion

The results of this study are very satisfactory, as well on the level of the validation of the formulations as on the level of the numerical programming by finite elements or the integral equations at the borders. However, the importance of this work finds its interest in possible cracking's which we often meet in the finished products of the composite structures .We stuck, during this work, to some extent to validate calculations resulting from the software finite elements, which we developed within our laboratory. Lastly, this work is integrated in a study in which we are interested in the localization zones of decoherence (fibre - matrix) which can, as we have just presented it, easily be analyzed by the techniques of the integral equations of borders and of shown the performances of this method in calculation of the rupture .

References

[1] H. CHAFFOUI, D. PLAY, 'European Review of the Finite elements', Study of the mechanical behavior for the textile structural design, October 1998, V 7, N 6, , pp.737 -754

[2] S. Alsubari, H. Chaffoui, Modelling and Composite Analysis of Laminated Structures, International Newspaper of Computer Applications (0975 - 8887) Volume 50 No.13, July 2012, pp. 1-5

[3] S. Alsubari, H. Chaffoui, Study of the Mechanical Behavior of Composite Punts by Homogenisation, sciencedirect, Procedia Engineering 15, 2011, Elsevier, pp. 4063 - 4067

[4] S. Alsubari, H. Chaffoui, Comparison of the elastic coefficients and Calculation Models of the

Mechanical Behavior one- Dimensional Composites, International IJSI Newspaper of Computer Science Exits, vol. 8, Issue 5, No 2, September 2011, pp. 63-67

[5] T.A. Cruse and F.J. Rizzo, Boundry-integral equation method computation application in applied mechanics, Proc. Am.Ploughshare.Mech.Eng., 11, 1975.

[6] P.K.Banerjee, R. Butterfiel, Boundry element methods in engineering Science, Mc Graw-Hill Book Company (1980).

[7] O.C. Zienkiewicz, finite element method, Mc Graw-Hill Book Company, 1979.

[8] J P. Henry, F Parsy, Course of elasticity, Bordered, Paris, 1982.

[9] N I Muskhelishvili, Some BASIC problems of the mathematical theory of elasticity, 4th edition, Noord hoff, 1963.

[10] H. CHAFFOUI, , 8th Congress of Mécaique, Study of the mechanical behavior of the composite plates, April 2007, El jadida, pp. 335-337

[11] R. Mr. Christensen, Mechanics of Composite Materials, ED John Wiley & Sounds, 1979.

[12] Zuorong Chen , Wenyi Yan A shear-lag model with a cohesive fibre–matrix interface for analysis of fibre pull-out , Mechanics of Materials 91 (2015), pp. 119–135

[13] A. Ríos and A. Martín-Meizoso , Micromechanical Model of Interface Between Fibre And Matrix of Metal Matrix Composite Reinforced With Continuous Fibre , Advanced materials research Vol. 59 (2009) ,pp. 158-163

[14] A. PAIPETIS, C. GALIOTIS and Y. CHING , J. A. NAIRN. Stress Transfer From the Matrix to the Fibre in a Fragmentation Test: Raman Experiments and Analytical Modeling. Journal of Composite Materials, Vol. 33, No. 4, 377, 399/1999.

[15] Mokhtar Khaldi, Alexandre Vivet, Alain Bourmaud, Zouaoui Sereir, Belkacem Kada. Damage analysis of composites reinforced with Alfa fibers: Viscoelastic behavior and debonding at the fiber/matrix interface . Article in Journal of applied polymer science. Applied polymer symposium · April 2016

[16] Johannes G , M. Schemmann , Thoma S, Andrew H. and Thomas B. Sensitivity Analysis of Fiber-Matrix Interface Parameters in an SMC Composite Damage Model , Published: Proceedings ,12 July 2018 .

[17] Alexia Este ,Bernard Toson, J. Saliba, A New Approach to Simulate Interface Damage in Brittle Matrix composites , Procedia Structural Integrity 2 (2016) ,pp. 2456–2462.

[18] Thomas Jollivet, Catherine Peyrac, Fabien Lefebvre , Damage of composite materials, Procedia Engineering 66 (2013),pp. 746 – 758.

[19] Enrique Graciani , Vladislav Mantic, Federico Parıs, Antonio Blazquez ,Weak formulation of axi-symmetric frictionless contact problems with boundary elements Application to interface cracks , Computers and Structures 83 (2005), pp. 836–855.

[20] Chuwei Zhou, Wei Yang and Daining Fang , A Numerical Strength Analysis of Metal Matrix Composites Considering the Interface and Matrix Damage Evolution, METALS AND MATERIALS, VoL 4, No. 4 (1998), pp. 624 -627.

[21] B. Sabuncuoglu , S. A. Tabatabaei and S.V. Lomov, European Conference on Composite Materials , Athens, Greece, Analysis of Fiber/Matrix Interface Debonding In Steel Fiber Composites Under Transverse Loading, ECCM18 - 18th, 24-28th June 2018.

[22] Roman Vodika, Jozef Ksinan, Interfacial Debonds in Unidirectional Fibre-Reinforced Composites Exposed to Biaxial Loads, Procedia Engineering, 190 (2017), pp. 433 – 440

[23] Alessandro Abena, Sien Leung Soo, Khamis Essa finite element simulation for orthogonal cutting of UD-CFRP incorporating a novel fibre – matrix interface model, Procedia CIRP 31 (2015), pp. 539 – 544

[24] Enrique Graciani, Vladislav Mantic, Federico Paris , Janis Varna, Numerical analysis of debond propagation in the single fibre fragmentation test, Composites Science and Technology 69 (2009), pp. 2514–2520

[25] Enrique Graciani , Vladislav Mantic, Federico Parıs , Antonio Blazquez , Weak formulation of axi-symmetric frictionless contact problems with boundary elements Application to interface cracks, Computers and Structures 83 (2005), pp. 836–855.

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